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OPERATING CHARACTERISTIC CURVES AND OTHER FEATURES OF TRUNCATED LIFE TESTS WITH REPLACEMENT

by
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WAYNE UNIVERSITY
DETROIT, MICHIGAN

OPERATING CHARACTERISTIC CURVES AND OTHER FEATURES OF

TRUNCATED LIFE TESTS WITH REPLACEMENT

by

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1. Summary

In [1] we considered the properties of truncated life tests under the assumption that the underlying life distribution is exponential. The purpose of this report is to furnish some tables and graphs for the replacement case when items which fail are replaced at once by new items.

The underlying probability density function of life is assumed throughout to be

$$(1) \qquad f(x;\theta) = \frac{1}{\theta} e^{-x/\theta}, \quad x > 0, \ \theta > 0.$$

The test is started with n items drawn at random from (1). Any item which fails is replaced at once by a new item drawn from (1). The experiment is truncated at time $T = \min(X_{r_o,n}, T_o)$, where $X_{r_o,n}$ is the time (measured from the beginning of the experiment) when the r_o'th failure occurs and T_o is a truncation time beyond which the experiment is not permitted to run.

2. Recapitulation of Necessary Formulae

We recall certain results proved in [1] concerning the truncated replacement situation. Briefly it was shown that $Pr(r=k|\theta)$, the probability of reaching a decision which requires exactly k failures is given by

(2) $$Pr(r=k|\theta) = p(k;\lambda_\theta), \qquad k = 0,1,2,\ldots,r_o-1$$

and

(3) $$Pr(r=r_o|\theta) = 1 - \sum_{k=1}^{r_o-1} p(k;\lambda_\theta),$$

where $p(k;\lambda_\theta) = \lambda_\theta^k e^{-\lambda_\theta}/k!$ and $\lambda_\theta = nT_o/\theta$.

While $E_\theta(r)$, the expected number of observations required to reach a decision can be written down at once from the definition of expectation, it is useful for computational reasons to use the formula

(4) $$E_\theta(r) = \lambda_\theta\left[\sum_{k=0}^{r_o-2} p(k;\lambda_\theta)\right] + r_o\left[1 - \sum_{k=0}^{r_o-1} p(k;\lambda_\theta)\right].$$

In this form one can use Molina's tables on the Poisson distribution [2]. Further $E_\theta(T)$, the expected waiting time to reach a decision, is related to $E_\theta(r)$, by the formula

(5) $$E_\theta(T) = \frac{\theta}{n} E_\theta(r).$$

A useful quantity to tabulate is the ratio $E_\theta(T)/T_o$ (this must obviously be ≤ 1). From the definition of λ_θ and (5) it follows at once that

(6) $$E_\theta(T)/T_o = E_\theta(r)/\lambda_\theta.$$

Suppose we use the following rule of action:

(i) If $\min(X_{r_o,n}, T_o) = X_{r_o,n}$ (i.e., r_o failures occur before time T_o), reject the hypothesis that $\theta = \theta_o$.

(ii) If $\min(X_{r_o,n}, T_o) = T_o$ (i.e., the r_o'th failure occurs after time T_o), accept the hypothesis that $\theta = \theta_o$. Then $L(\theta)$(1), the probability of accepting $\theta = \theta_o$ when θ is the true value, is simply

$$L(\theta) = \sum_{k=0}^{r_o-1} p(k; \lambda_\theta). \tag{7}$$

It is convenient to express various features of the test procedures in terms of the dimensionless parameter $\lambda = nT_o/\theta$. In Table 1 we give $L(\lambda)$, $E_\lambda(r)$, and $E_\lambda(T)/T_o$ for $r_o = 1(1)20(5)100$. Graphs of each of these functions are given for $r_o = 1(1)10(5)25$ in Figures 1,2, and 3. The tables and figures are useful in all cases where r_o, T_o, and n are preassigned. The extension of the tables beyond the values tabulated can be easily performed, since for sufficiently small λ, $E_\lambda(r) \sim \lambda$ and $E_\lambda(T)/T_o \sim 1$. For sufficiently large λ, $E_\lambda(r) = r_o$, and $E_\lambda(T)/T_o = r_o/\lambda$.

It often happens in practice that it is important to preassign the truncation time T_o. This involves finding a truncated replacement test (i.e., a suitable r_o and n) whose operating characteristic curve is such that $L(\theta_o) = 1-\alpha$ and $L(\theta_1) \leq \beta (\theta_o > \theta_1)$. Now it can easily be shown that the

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(1) $L(\theta)$ is the ordinate of the operating characteristic curve of the test procedure.

best[2] acceptance region (of size α) for H_o based on the first r out of n failures, is for preassigned r and n, given by[3]

(8) $$X_{r,n} > C = \theta_o \chi^2_{1-\alpha}(2r)/2n.$$

In order that the test based on this acceptance region have an operating characteristic curve for which $L(\theta_o) = 1-\alpha$ and $L(\theta_1) \leq \beta$, we need to choose r suitably. The appropriate values for r (which we call r_o) are given in Table 2. A decision rule based on (8) is clearly truncated with truncation time C. In order to truncate experimentation at $\min[X_{r_o,n}, T_o]$ it is necessary to choose n suitably. For all practical purposes one can choose n as

(9) $$n = \left[\theta_o \chi^2_{1-\alpha}(2r_o)/2T_o\right]$$

where [x] means the greatest integer $\leq$ x. If this value of n is used, it is easy to verify that using $X_{r_o,n}$ as the acceptance region for H_o will result in having $L(\theta_o) \geq 1-\alpha$, but $L(\theta_1)$ might in some cases be slightly $> \beta$.[4] It is interesting to note that the appropriate n (for fixed α and β) is

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(2) In the sense of Neyman and Pearson.

(3) We denote a chi-square variable with n degrees of freedom as χ^2_n and define η such that $\Pr(\chi^2_n > \eta) = \gamma$ as $\eta = \chi^2_\gamma(n)$.

(4) This could be avoided easily by giving the experimenter the freedom to use instead of T_o, the slightly larger truncation time $T''_o = \theta_o \chi^2_{1-\alpha}(2r_o)/n$, where n is given by (9). The test based on using $X_{r_o,n} > T''_o$ as acceptance region for H_o will have $L(\theta_o) = 1-\alpha$ and $L(\theta_1) \leq \beta$. If one uses n+1 items throughout the test, then one could use instead of T_o, the slightly smaller truncation time $T'_o = \theta_o \chi^2_{1-\alpha}(2r_o)/(n+1)$. The test based on using $X_{r_o,n+1} > T'_o$ as acceptance region for H_o will have $L(\theta_o) = 1-\alpha$ and $L(\theta_1) \leq \beta$.

inversely proportional to the time of truncation, T_o. Thus, e.g., to reduce the truncation time by a factor of two requires doubling n.

3. Numerical Examples

Problem 1. A sample of 20 tubes is drawn at random from a lot whose life is assumed to follow the exponential density (1). Tubes that fail are replaced at once by new tubes drawn from the lot. The following rule of action is followed: If 2 failures occur before 500 hours have elapsed, the test is stopped as soon as the second failure has occurred and the lot is rejected. If, however, 500 hours have elapsed before 2 failures have occurred, the test is stopped at 500 hours and the lot is accepted.

Question 1: What is the probability of accepting a lot whose mean life $\theta = 2000$ hours? What is the expected number of hours required to make the decision if $\theta = 2000$? How many items will be failed on the average if $\theta = 2000$? Using Table 1 and noting that $n = 20$, $r_o = 2$, and $T_o = 500$, it is clear that $\lambda = nT_o/\theta = 5$. Hence the probability of accepting a lot with $\theta = 2000$ is .04. The expected waiting time to reach a decision is 185 hours and the expected number of failures is 1.95.

Question 2: Same as question 1 for $\theta = 10{,}000$ hours. In this case $\lambda = 1$. The probability of accepting the lot is .736, the expected waiting time is 448 hours, and the expected number of items failed before reaching a decision is .896.

Question 3: How large should θ be in order that there be a probability that a lot with mean life θ is accepted 95% of the time? It is easy to compute that this requires $\lambda = .355$. Hence from the relation $\lambda = nT_o/\theta$, it follows that $\theta = 10{,}000/.355 = 28{,}170$. Hence a lot with a mean life of 28,170 is accepted 95% of the time by this plan.

Question 4: Same as 3, except that we want the lot rejected 95% of the time. This requires $\lambda = 4.75$ and $\theta = 2105$ hours. Hence a lot having a mean life of 2105 hours will be rejected by this plan 95% of the time.

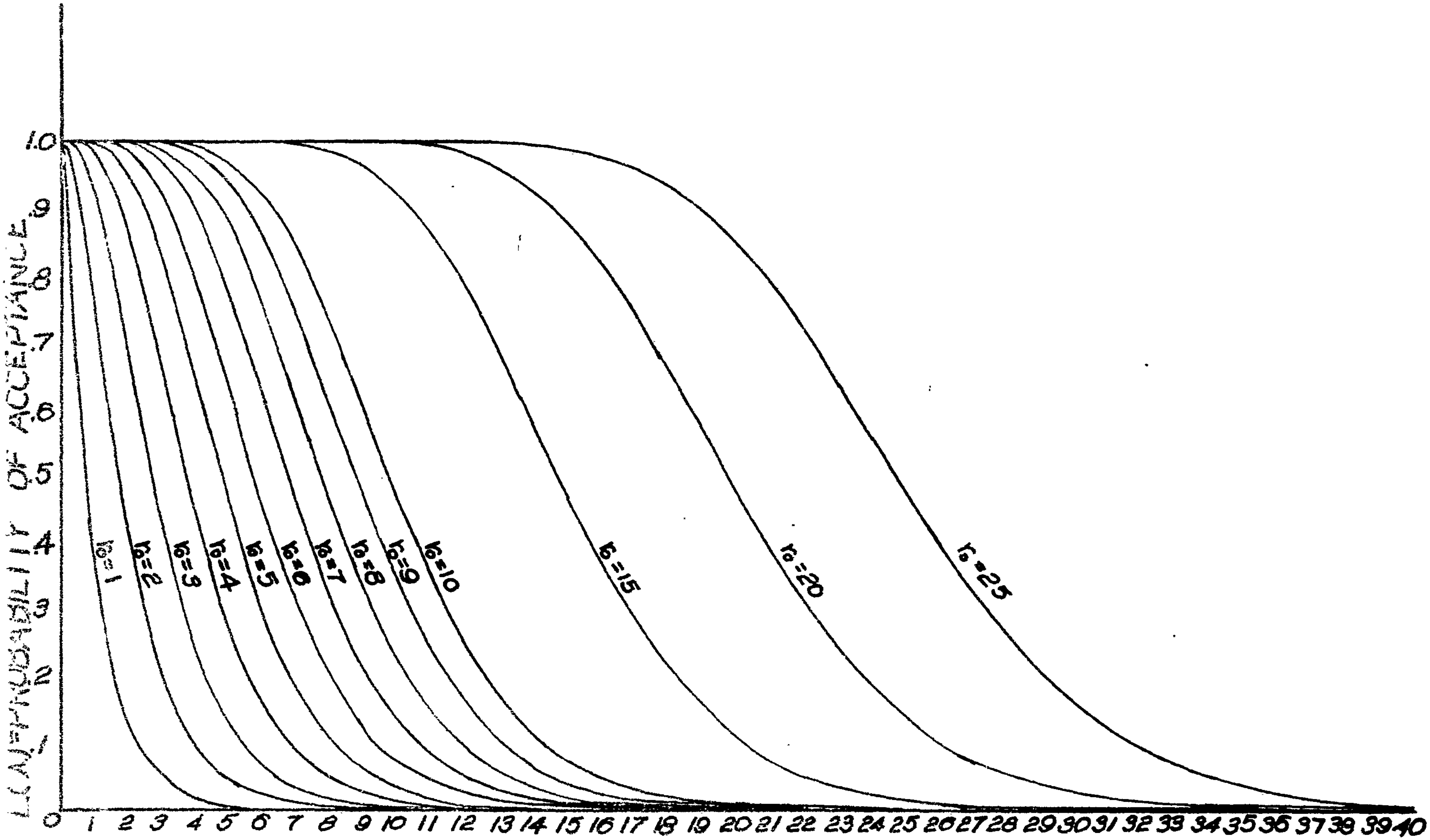
Problem 2. Find a truncated replacement plan for which $T_o = 200$, and which will accept a lot with a mean life of 3000 hours, 90% of the time, and reject a lot with a mean life of 1000 hours, 90% of the time. In this case $\theta_o = 3000$, $\theta_1 = 1000$, $\alpha = \beta = .1$. Since $\theta_o/\theta_1 = 3$, it follows from Table 2 that $r_o = 6$. Consequently $n = \left[\theta_o \chi^2_{.9}(12)/400\right] = \left[\frac{(3000)(6.304)}{400}\right] = 47$.

Thus the following truncated replacement plan should be used: Start with $n = 47$ items. As soon as any item fails, replace it by a new one. Accept the lot if $\min\left[X_{6,47}, 200\right] = 200$ and reject the lot if $\min\left[X_{6,47}, 200\right] = X_{6,47}$. If the lot is rejected experimentation is stopped at $X_{6,47}$, the time of occurrence of the sixth failure. It is readily verified that this plan meets the conditions specified.

REFERENCES

1. B. Epstein, "Some Results on Truncated Life Tests in the Exponential Case". Submitted for publication. Also issued as Technical Report No.4.

2. E. C. Molina, "Poisson's Exponential Binomial Limit", Tables I and II, D. Van Nostrand Co., 1949.



$\lambda = nT_0/\theta$

FIGURE 1: OPERATING CHARACTERISTIC CURVE OF TRUNCATED REPLACEMENT LIFE TEST

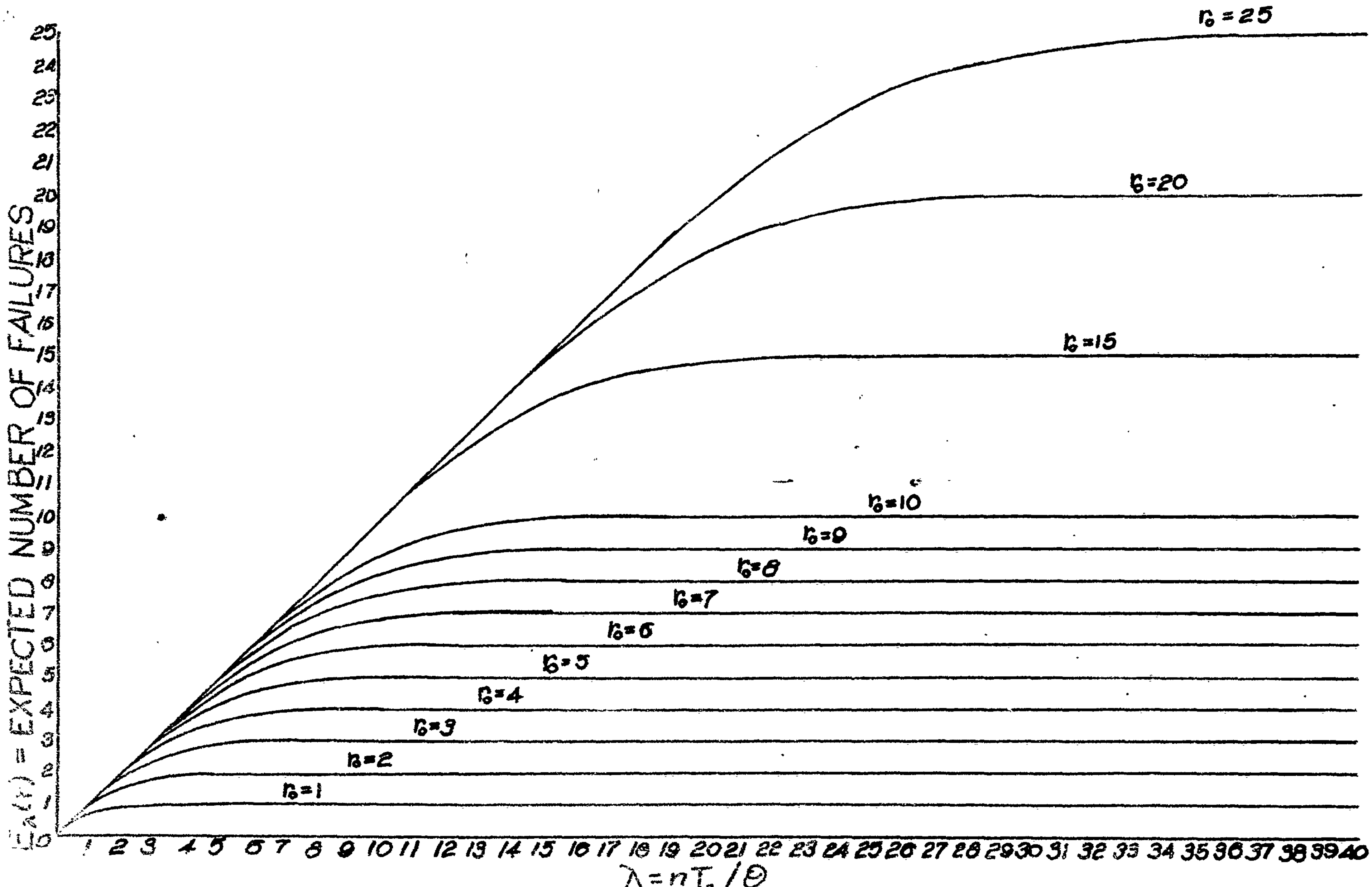


FIGURE 2: EXPECTED NUMBER OF FAILURES REQUIRED TO REACH A DECISION IN TRUNCATED REPLACEMENT LIFE TESTS

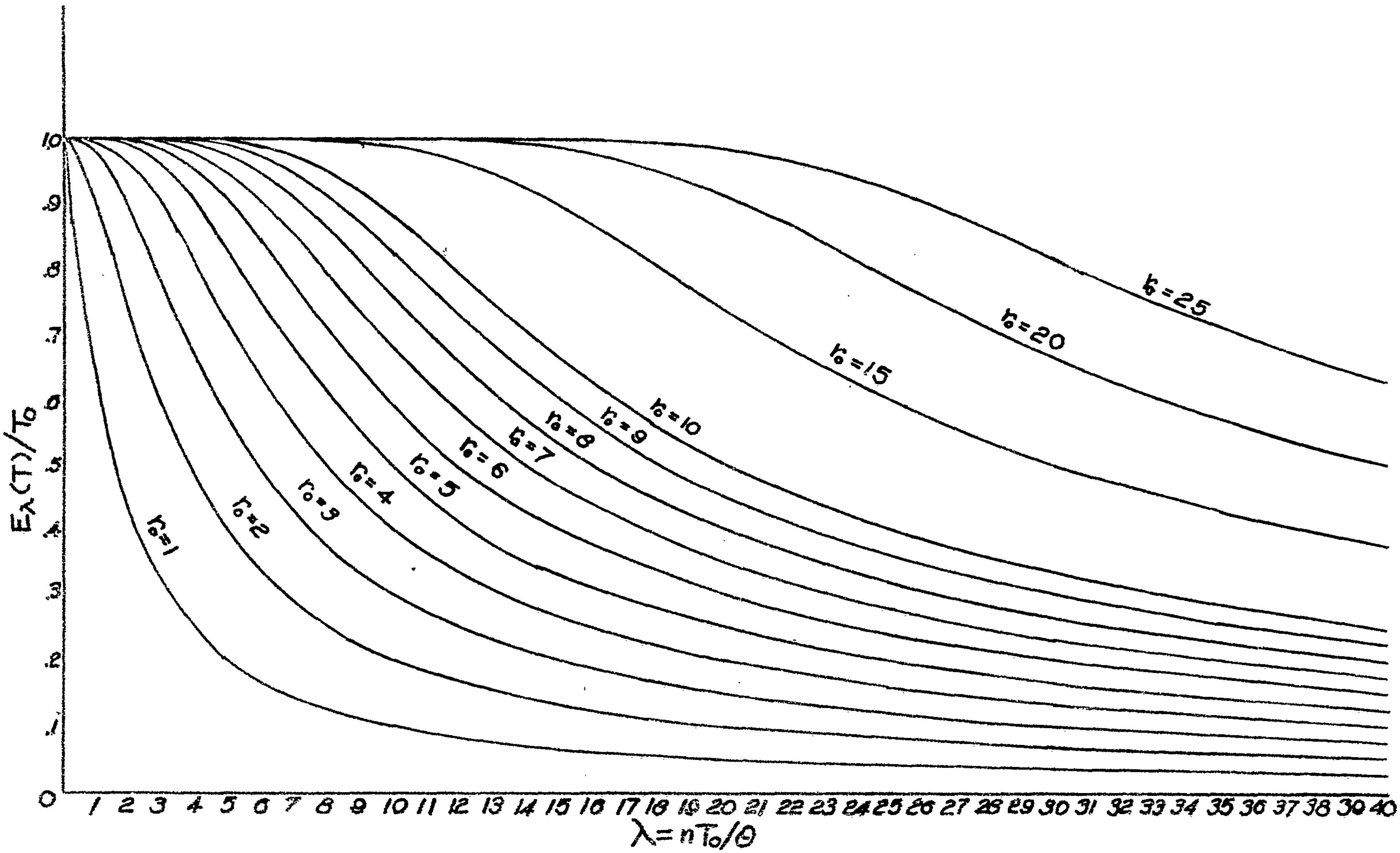


FIGURE 3: EXPECTED WAITING TIME TO REACH A DECISION IN TRUNCATED REPLACEMENT LIFE TESTS

Table 1

Characteristics of Truncated Replacement Procedures in the Exponential Case.*

$r_o = 1$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
.005	.995	.005	.998
.01	.990	.010	.995
.05	.951	.049	.975
.1	.905	.095	.952
.2	.819	.181	.906
.3	.741	.259	.864
.4	.670	.330	.824
.5	.607	.393	.787
.7	.497	.503	.719
1.0	.368	.632	.632
1.5	.223	.777	.518
2.0	.135	.865	.432
3.0	.050	.950	.317
4.0	.018	.982	.245
5.0	.007	.993	.199
10.0	.000	1.000	.100

* The description of the test procedure is given in Sections 1 and 2 of this report. The definitions of r_o, T_o, λ, and formulae for $L(\lambda)$, $E_\lambda(r)$, and $E_\lambda(T)/T_o$ are given in Section 2. A more detailed development of the underlying mathematics is given in [1].

$r_o = 2$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
.01	1.000	.010	1.000
.1	.995	.100	.998
.2	.982	.199	.994
.3	.963	.296	.987
.4	.938	.391	.978
.5	.910	.484	.967
.7	.844	.659	.942
1.0	.736	.896	.896
1.5	.558	1.230	.820
2.0	.406	1.459	.729
2.5	.287	1.631	.652
3.0	.199	1.751	.584
4.0	.092	1.890	.473
5.0	.040	1.953	.391
10.0	.000	1.999	.200

$r_o = 3$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
.1	1.000	.100	1.000
.3	.996	.300	.999
.5	.986	.498	.996
.7	.966	.693	.991
1.0	.920	.977	.977
1.5	.809	1.410	.940
2.0	.677	1.782	.891
2.5	.544	2.087	.835
3.0	.423	2.328	.776
3.5	.321	2.513	.718
4.0	.238	2.652	.663
5.0	.125	2.828	.566
7.0	.030	2.962	.423
10.0	.003	2.997	.300

$r_o = 4$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
.2	1.000	.200	1.000
.5	.998	.500	1.000
.7	.994	.699	.999
1.0	.981	.996	.996
1.5	.934	1.476	.984
2.0	.857	1.925	.962
2.5	.758	2.329	.932
3.0	.647	2.681	.894
3.5	.537	2.976	.850
4.0	.433	3.219	.805
4.5	.342	3.412	.758
5.0	.265	3.563	.713
6.0	.151	3.767	.628
7.0	.082	3.880	.554
8.0	.042	3.941	.493
9.0	.021	3.971	.441
10.0	.010	3.986	.399
12.0	.002	3.997	.333
15.0	.000	4.000	.267

$r_o = 5$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
.5	1.000	.500	1.000
1.0	.997	.999	.999
1.5	.981	1.494	.996
2.0	.941	1.983	.991
2.5	.891	2.438	.975
3.0	.815	2.864	.955
3.5	.725	3.251	.929
4.0	.629	3.590	.897
4.5	.532	3.880	.862
5.0	.440	4.123	.825
5.5	.358	4.322	.786
6.0	.285	4.482	.747
7.0	.173	4.707	.672
8.0	.100	4.841	.605
9.0	.055	4.916	.546
10.0	.029	4.957	.496
15.0	.001	5.000	.333
20.0	.000	5.000	.250

$r_o = 6$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
.5	1.000	.500	1.000
1.0	.999	1.000	1.000
2.0	.983	1.994	.997
3.0	.916	2.949	.983
3.5	.858	3.393	.970
4.0	.785	3.805	.951
4.5	.703	4.177	.928
5.0	.616	4.507	.901
5.5	.529	4.793	.871
6.0	.446	5.036	.839
6.5	.369	5.240	.806
7.0	.301	5.407	.772
8.0	.191	5.650	.706
9.0	.116	5.801	.645
10.0	.067	5.890	.589
15.0	.003	5.998	.400
20.0	.000	6.000	.300

$r_o = 7$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
1.0	1.000	1.000	1.000
2.0	.995	1.999	.999
3.0	.966	2.983	.994
4.0	.889	3.915	.979
5.0	.762	4.745	.949
5.5	.686	5.107	.929
6.0	.606	5.430	.905
6.5	.527	5.713	.879
7.0	.450	5.957	.851
8.0	.313	6.336	.792
9.0	.207	6.594	.733
10.0	.130	6.760	.676
12.0	.046	6.923	.577
15.0	.008	6.990	.466
20.0	.000	7.000	.350

$r_o = 8$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
1.0	1.000	1.000	1.000
2.0	.999	2.000	1.000
3.0	.988	2.995	.998
4.0	.949	3.966	.992
5.0	.867	4.877	.975
6.0	.744	5.821	.970
6.5	.673	5.980	.920
7.0	.599	6.358	.908
7.5	.525	6.639	.885
8.0	.453	6.883	.860
9.0	.324	7.270	.808
10.0	.220	7.540	.754
11.0	.143	7.719	.702
12.0	.090	7.834	.653
13.0	.054	7.904	.608
14.0	.032	7.946	.568
15.0	.018	7.972	.531
17.0	.005	7.992	.470
20.0	.001	7.999	.400
25.0	.000	8.000	.320

$r_o = 9$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
2.0	1.000	2.000	1.000
3.0	.996	2.999	1.000
4.0	.979	3.988	.997
5.0	.932	4.946	.989
6.0	.847	5.839	.973
6.5	.792	6.249	.961
7.0	.729	6.629	.947
7.5	.662	6.977	.930
8.0	.593	7.291	.911
8.5	.523	7.570	.891
9.0	.456	7.818	.869
10.0	.333	8.207	.821
11.0	.232	8.487	.772
12.0	.155	8.679	.723
13.0	.100	8.805	.677
14.0	.062	8.884	.635
15.0	.037	8.933	.596
17.0	.013	8.979	.528
20.0	.002	8.997	.450
25.0	.000	9.000	.360

$r_o = 10$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
2.0	1.000	2.000	1.000
3.0	.999	3.000	1.000
4.0	.992	3.996	.999
5.0	.968	4.978	.996
6.0	.916	5.923	.987
7.0	.830	6.799	.971
8.0	.717	7.574	.947
8.5	.653	7.917	.931
9.0	.587	8.227	.914
10.0	.458	8.749	.875
11.0	.341	9.147	.832
12.0	.242	9.436	.786
13.0	.166	9.639	.741
14.0	.109	9.775	.698
15.0	.070	9.863	.658
17.0	.026	9.953	.585
20.0	.005	9.992	.500
25.0	.000	10.000	.400

$r_o = 11$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
2.0	1.000	2.000	1.000
3.0	1.000	3.000	1.000
4.0	.997	3.999	1.000
5.0	.986	4.992	.998
6.0	.957	5.965	.994
7.0	.901	6.897	.985
8.0	.816	7.763	.970
9.0	.706	8.521	.947
10.0	.583	9.166	.917
11.0	.460	9.687	.881
12.0	.347	10.089	.841
13.0	.252	10.387	.799
14.0	.176	10.599	.757
15.0	.118	10.745	.716
17.0	.049	10.904	.641
20.0	.011	10.981	.549
25.0	.001	10.999	.440
30.0	.000	11.000	.367

$r_o = 12$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
3.0	1.000	3.000	1.000
4.0	.999	4.000	1.000
5.0	.995	4.997	.999
6.0	.980	5.985	.998
7.0	.947	6.951	.993
8.0	.888	7.870	.984
9.0	.803	8.718	.969
10.0	.697	9.469	.947
11.0	.579	10.108	.919
12.0	.462	10.628	.886
13.0	.353	11.034	.849
14.0	.260	11.339	.810
15.0	.185	11.560	.771
17.0	.085	11.817	.695
20.0	.021	11.960	.598
25.0	.001	11.998	.480
30.0	.000	12.000	.400

[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]
1	.000	.000	1.000
[illegible]	998	.999	1.000
[illegible]	[illegible]	5.994	.999
[illegible]	973	6.978	.997
7	936	7.934	.992
[illegible]	.876	8.812	.982
	.792	9.678	.963
10	.689	10.419	.947
11	.576	11.052	.921
12	.463	11.571	.890
13	.358	11.982	.856
14	.268	12.292	.839
15	.135	12.684	.746
17	.039	12.921	.646
20	.003	12.995	.520
25	.000	13.000	.432
30			

$r_o = 13$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
4	1.000	4.000	1.000
5	.998	4.999	1.000
6	.991	5.994	.999
7	.973	6.978	.997
8	.936	7.934	.992
9	.876	8.842	.982
10	.792	9.678	.968
11	.689	10.419	.947
12	.576	11.052	.921
13	.463	11.571	.890
14	.358	11.982	.856
15	.268	12.292	.819
17	.135	12.684	.746
20	.039	12.921	.646
25	.003	12.995	.520
30	.000	13.000	.433

$r_o = 14$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
4	1.000	4.000	1.000
5	.999	5.000	1.000
6	.996	5.998	1.000
7	.987	6.990	.999
8	.966	7.968	.996
9	.926	8.916	.991
10	.864	9.813	.981
11	.781	10.638	.967
12	.682	11.370	.948
13	.573	11.998	.923
14	.464	12.516	.894
15	.363	12.929	.862
16	.275	13.247	.828
17	.201	13.483	.793
18	.143	13.654	.759
19	.098	13.773	.725
20	.066	13.854	.693
25	.006	13.988	.560
30	.000	13.999	.467

$r_o = 15$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
5	1.000	5.000	1.000
6	.999	5.999	1.000
7	.994	6.996	.999
8	.983	7.985	.998
9	.959	8.957	.995
10	.917	9.897	.990
11	.854	10.784	.980
12	.772	11.598	.967
13	.675	12.323	.948
14	.570	12.946	.925
15	.466	13.463	.898
16	.368	13.879	.867
17	.281	14.202	.835
18	.208	14.446	.803
19	.150	14.623	.770
20	.105	14.750	.737
22	.048	14.896	.677
25	.012	14.974	.599
30	.001	14.998	.500
35	.000	15.000	.429

$r_o = 16$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
5	1.000	5.000	1.000
6	.999	6.000	1.000
7	.998	6.998	1.000
8	.992	7.994	.999
9	.978	8.979	.998
10	.951	9.945	.995
11	.907	10.876	.989
12	.844	11.754	.979
13	.764	12.559	.966
14	.669	13.276	.934
15	.568	13.895	.926
16	.467	14.413	.901
17	.371	14.831	.872
18	.287	15.159	.842
19	.215	15.409	.811
20	.157	15.593	.780
22	.077	15.819	.719
25	.022	15.953	.638
27	.009	15.983	.592
30	.002	15.996	.533
35	.000	16.000	.457

$r_0 = 17$

$\lambda = nT_0/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_0$
6	1.000	6.000	1.000
7	.999	6.999	1.000
8	.996	7.997	1.000
9	.989	8.990	.999
10	.973	9.972	.997
11	.944	10.932	.994
12	.899	11.856	.988
13	.835	12.724	.979
14	.756	13.520	.966
15	.666	14.197	.946
16	.566	14.847	.928
17	.468	15.363	.904
18	.375	15.784	.877
19	.292	16.117	.848
20	.221	16.372	.819
22	.117	16.702	.759
25	.038	16.916	.677
27	.016	16.967	.628
30	.004	16.993	.566
35	.000	17.000	.486

$r_o = 18$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
7	1.000	7.000	1.000
8	.998	7.999	1.000
9	.995	8.996	1.000
10	.986	9.987	.999
11	.968	10.964	.997
12	.937	11.918	.993
13	.890	12.832	.987
14	.827	13.693	.978
15	.749	14.512	.967
16	.659	15.187	.949
17	.564	15.799	.929
18	.469	16.315	.906
19	.378	16.738	.881
20	.297	17.075	.854
21	.227	17.336	.826
22	.169	17.533	.797
23	.123	17.678	.769
24	.087	17.782	.741
25	.060	17.930	.717
27	.027	17.939	.664
30	.007	17.985	.600
35	.001	17.999	.514
40	.000	18.000	.450

$r_o = 19$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
7	1.000	7.000	1.000
8	.999	8.000	1.000
9	.998	8.998	1.000
10	.993	9.994	.999
11	.982	10.982	.998
12	.963	11.955	.996
13	.930	12.903	.993
14	.883	13.811	.986
15	.819	14.663	.978
16	.742	15.445	.965
17	.655	16.144	.950
18	.562	16.753	.931
19	.469	17.269	.909
20	.381	17.694	.885
21	.302	18.034	.859
22	.232	18.300	.832
23	.175	18.503	.804
24	.128	18.654	.777
25	.092	18.763	.751
27	.044	18.895	.700
30	.013	18.972	.632
35	.001	18.998	.543
40	.000	19.000	.475

$r_o = 20$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
8	1.000	8.000	1.000
9	.999	8.999	1.000
10	.997	9.997	1.000
11	.991	10.991	.999
12	.979	11.977	.998
13	.957	12.946	.996
14	.923	13.887	.992
15	.875	14.788	.986
16	.812	15.633	.977
17	.736	16.408	.965
18	.651	17.102	.950
19	.561	17.708	.932
20	.470	18.223	.911
21	.384	18.650	.888
22	.306	18.994	.863
23	.238	19.265	.838
24	.180	19.474	.811
25	.134	19.630	.785
27	.069	19.826	.734
30	.022	19.951	.665
35	.002	19.995	.571
40	.000	20.000	.500

$r_o = 25$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
11	1.000	11.000	1.000
12	.999	11.999	1.000
13	.998	12.998	1.000
14	.995	13.995	1.000
15	.989	14.987	.999
16	.978	15.971	.998
17	.959	16.940	.996
18	.932	17.887	.994
19	.893	18.800	.989
20	.843	19.669	.983
21	.782	20.483	.975
22	.712	21.230	.965
23	.635	21.904	.952
24	.554	22.498	.937
25	.473	23.012	.920
26	.396	23.446	.902
27	.324	23.806	.882
28	.260	24.097	.861
29	.204	24.328	.839
30	.157	24.508	.817
35	.032	24.918	.712
40	.004	24.990	.625
45	.000	24.999	.556

$r_n = 30$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
15	1.000	15.000	1.000
16	.999	15.999	1.000
17	.997	16.997	1.000
18	.994	17.993	1.000
19	.988	18.984	.999
20	.978	19.968	.998
21	.963	20.939	.997
22	.940	21.891	.995
23	.908	22.815	.992
24	.868	23.704	.988
25	.818	24.548	.982
26	.759	25.337	.975
27	.693	26.054	.965
28	.623	26.723	.954
29	.549	27.309	.942
30	.476	27.821	.927
31	.405	28.261	.912
32	.338	28.632	.895
33	.277	28.939	.877
34	.224	29.189	.858
35	.177	29.388	.831
37	.106	29.666	.802
40	.043	29.878	.747
45	.007	29.982	.666
50	.001	29.998	.600
60	.000	30.000	.500

$r_o = 35$

$\lambda = \pi r_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
20	.999	19.998	1.000
22	.994	21.991	1.000
25	.966	24.940	.998
26	.947	25.897	.996
27	.921	26.832	.994
28	.888	27.738	.991
29	.847	28.598	.986
30	.797	29.428	.981
31	.741	30.197	.974
32	.679	30.908	.966
33	.613	31.555	.956
34	.545	32.134	.945
35	.478	32.645	.933
36	.411	33.090	.919
37	.349	33.470	.905
38	.291	33.789	.889
39	.240	34.054	.873
40	.194	34.270	.857
42	.121	34.582	.823
45	.054	34.834	.774
50	.011	34.972	.699
60	.000	35.000	.583

$r_o = 40$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_{\lambda}(r)$	$E_{\lambda}(T)/T_o$
25	.997	24.995	1.000
27	.989	26.982	.999
30	.954	29.905	.997
32	.904	31.767	.993
35	.780	34.309	.980
36	.726	35.063	.974
37	.668	35.760	.966
38	.606	36.397	.958
39	.542	36.971	.948
40	.479	37.482	.937
41	.417	37.930	.925
42	.358	38.317	.912
43	.303	38.647	.899
44	.253	38.925	.885
45	.208	39.155	.870
46	.169	39.344	.855
47	.136	39.496	.840
48	.107	39.617	.825
49	.084	39.712	.810
50	.065	39.786	.796
55	.015	39.958	.727
60	.003	39.994	.667
70	.000	40.000	.571

$r_o = 45$

$\lambda = r\lambda_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
30	.994	29.990	1.000
35	.941	34.864	.996
37	.889	36.698	.992
40	.766	39.193	.980
42	.658	40.620	.967
44	.540	41.819	.950
46	.422	42.780	.930
48	.313	43.512	.907
50	.221	44.043	.881
52	.148	44.409	.854
54	.095	44.650	.827
56	.058	44.800	.800
58	.034	44.891	.774
60	.019	44.942	.749
65	.004	44.990	.692
70	.001	44.999	.643
80	.000	45.000	.563

$r_o = 50$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
30	.999	29.999	1.000
35	.990	34.982	.999
40	.930	39.818	.995
42	.875	41.626	.991
44	.799	43.304	.984
46	.703	44.808	.974
48	.595	46.108	.961
50	.481	47.184	.944
52	.372	48.036	.924
54	.275	48.680	.901
56	.194	49.146	.878
58	.131	49.468	.853
60	.084	49.680	.828
65	.024	49.924	.768
70	.005	49.985	.714
80	.000	50.000	.625

$r_o = 55$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
35	.999	34.998	1.000
40	.986	39.971	.999
45	.918	44.769	.995
47	.862	46.552	.990
50	.742	48.969	.979
52	.643	50.357	.968
54	.536	51.537	.954
56	.429	52.501	.938
58	.329	53.258	.918
60	.242	53.827	.897
62	.171	54.237	.875
64	.116	54.521	.852
66	.075	54.709	.829
68	.047	54.829	.806
70	.028	54.903	.784
75	.007	54.979	.733
80	.001	54.996	.687
90	.000	55.000	.611

$r_o = 60$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
35	1.000	35.000	1.000
40	.998	39.997	1.000
45	.981	44.958	.999
50	.908	49.716	.994
52	.851	51.478	.990
54	.776	53.107	.983
56	.686	54.571	.974
58	.586	55.845	.963
60	.483	56.914	.949
62	.383	57.779	.932
64	.292	58.451	.913
66	.232	58.955	.893
68	.151	59.317	.872
70	.102	59.568	.851
72	.067	59.735	.830
75	.033	59.880	.798
80	.009	59.972	.750
90	.000	59.9[illegible]	.667

$r_o = 65$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
40	1.000	40.000	1.000
45	.997	44.995	1.000
50	.976	49.942	.999
55	.898	54.662	.994
60	.724	58.757	.979
62	.632	60.114	.970
64	.533	61.279	.957
66	.435	62.247	.943
68	.342	62.427	.918
70	.259	63.620	.909
72	.189	64.067	.890
74	.134	64.388	.870
76	.091	64.610	.850
78	.060	64.759	.830
80	.038	64.855	.811
85	.011	64.964	.764
90	.002	64.993	.722
100	.000	65.000	.650

$r_o = 70$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
40	1.000	40.000	1.000
45	1.000	44.999	1.000
50	.996	49.991	1.000
55	.971	54.924	.999
60	.888	59.606	.993
62	.830	61.327	.989
64	.758	62.917	.983
66	.673	64.349	.975
68	.580	65.602	.965
70	.484	66.666	.952
72	.391	67.541	.938
74	.305	68.235	.922
76	.231	68.769	.905
78	.168	69.166	.887
80	.119	69.450	.868
85	.043	69 829	.822
90	.013	69.955	.777
95	.003	69.990	.737
100	.001	69.998	.700

$r_o = 75$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
45	1.000	45.000	1.000
50	.999	49.999	1.000
55	.994	54.987	1.000
60	.966	59.903	.998
65	.879	64.548	.993
70	.710	68.454	.978
72	.623	69.887	.971
74	.531	71.041	.960
76	.439	72.010	.948
78	.352	72.800	.933
80	.273	73.424	.918
82	.205	73.900	.901
84	.150	74.253	.884
86	.105	74.506	.866
88	.072	74.682	.849
90	.048	74.801	.831
95	.015	74.945	.789
100	.004	74.987	.750
110	.000	75.000	.682

$r_o = 80$

$\lambda = nr_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
50	1.000	50.000	1.000
55	.999	54.998	1.000
60	.992	59.981	1.000
65	.961	64.860	.998
70	.871	69.490	.993
72	.813	71.176	.989
74	.742	72.733	.983
76	.662	74.138	.976
78	.575	75.376	.966
80	.485	76.436	.955
82	.398	77.318	.943
84	.317	78.031	.929
86	.245	78.590	.914
88	.183	79.016	.898
[illegible]	[illegible]	79.331	.882
95	[illegible]	79.771	.840
100	.014	[illegible].933	[illegible]
110	.002	[illegible].000	[illegible]
[illegible]	.000	[illegible]	[illegible]

$r_o = 85$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
55	1.000	55.000	1.000
60	.999	59.997	1.000
65	.990	64.975	1.000
70	.955	69.855	.998
74	.887	73.555	.994
76	.836	75.280	.991
78	.772	76.889	.986
80	.697	78.360	.980
82	.615	79.674	.972
84	.529	80.818	.962
86	.443	81.790	.951
88	.360	82.592	.939
90	.285	83.236	.925
92	.219	83.738	.910
94	.164	84.119	.895
96	.119	84.400	.879
98	.084	84.601	.863
100	.058	84.741	.847
105	.023	84.953	.809
110	.008	84.994	.773
125	.000	84.991	.680

$r_o = 90$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
60	1.000	60.000	1.000
65	.998	64.996	1.000
70	.988	69.967	1.000
75	.950	74.829	.998
80	.856	79.371	.992
82	.798	81.026	.988
84	.730	82.555	.983
86	.653	83.939	.976
88	.570	85.162	.968
90	.486	86.219	.958
92	.403	87.108	.947
94	.326	87.836	.934
96	.257	88.417	.921
98	.196	88.869	.907
100	.146	89.210	.892
105	.065	89.773	.855
110	.025	89.941	.818
125	.001	90.000	.720

$r_o = 95$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
65	1.000	65.000	1.000
70	.997	69.994	1.000
75	.985	74.958	.999
80	.944	79.801	.998
82	.914	81.661	.996
84	.873	83.449	.993
86	.821	85.146	.990
88	.759	86.727	.986
90	.687	88.174	.980
92	.609	89.471	.973
94	.527	90.608	.964
96	.446	91.581	.954
98	.367	92.393	.943
100	.295	93.055	.931
105	.153	94.229	.897
110	.071	94.664	.861
125	.003	95.000	.760
150	.000	95.000	.633

$r_o = 100$

$\lambda = nT_o/\theta$	$L(\lambda)$	$E_\lambda(r)$	$E_\lambda(T)/T_o$
70	1.000	69.999	1.000
75	.997	74.992	1.000
80	.983	79.948	.999
85	.939	84.771	.997
90	.842	89.250	.992
92	.785	90.878	.988
94	.719	92.383	.983
96	.645	93.748	.977
98	.567	94.961	.969
100	.487	96.015	.960
105	.296	98.035	.934
110	.158	99.169	.902
125	.011	99.983	.800
150	.000	100.000	.667

Table 2

Values of r_o needed to meet the condition that the test based on using $X_{r_o,n} > C$ as acceptance region in the replacement case will have the property that $L(\theta_o) = 1-\alpha$ and $L(\theta_1) \leq \beta$

$\alpha = .01$

θ_o/θ_1	$\beta = .01$	$\beta = .05$	$\beta = .10$
3/2	136	101	83
2	46	35	30
5/2	27	21	18
3	19	15	13
4	12	10	9
5	9	8	7
10	5	4	4

$\alpha = .05$

θ_o/θ_1	$\beta = .01$	$\beta = .05$	$\beta = .10$
3/2	95	67	55
2	33	23	19
5/2	19	14	11
3	13	10	8
4	9	7	6
5	7	5	4
10	4	3	3

$\alpha = .10$

θ_o/θ_1	$\beta = .01$	$\beta = .05$	$\beta = .10$
3/2	77	52	41
2	26	18	15
5/2	15	11	9
3	11	8	6
4	7	5	4
5	5	4	3
10	3	2	2